CAPÍTULO 1

1)

a) $\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$

b) $\alpha\frac{\partial^2 T}{\partial x^2}+\frac{q'''}{\rho c_p}=\frac{\partial T}{\partial t}$

c) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}=0$

d) $\alpha\frac{d^2 T}{dz^2}=v_z\frac{dT}{dz}$

e) $\frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q'''}{\rho c_p}=\frac{\partial T}{\partial t}$

2)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

C.C.1: $T(0,y)=T_o$

C.C.2: $-k\frac{\partial T(L,y)}{\partial x}=h\left[T(L,y)-T_\infty\right]$

C.C.3: $\frac{\partial T(x,0)}{\partial y}=0$

C.C.4: $-k\frac{\partial T(x,W)}{\partial y}=q''_o$

3)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{q'''}{k}=0$$

C.C.1: $T(r_i)=T_i$

C.C.2: $-k\frac{dT(r_o)}{dr}=-q''_o$

4)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

C.C.1: $-k\dfrac{\partial T(x,0)}{\partial y} = q_o''$

C.C.2: $-k\dfrac{\partial T(x,W)}{\partial y} = h\left[T(x,W) - T_\infty\right]$

C.C.3: $-k\dfrac{\partial T(0,y)}{\partial x} = h\left[T_\infty - T(0,y)\right]$

C.C.4: $\dfrac{\partial T(L,y)}{\partial x} = 0$

5)

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0$$

C.C.1: $-k\dfrac{\partial T(R_i,\theta)}{\partial r} = h_i\left[T_i - T(R_i,\theta)\right]$

C.C.2: $-k\dfrac{\partial T(R_o,\theta)}{\partial r} = h_o\left[T(R_o,\theta) - T_o\right]$

C.C.3: $T(r,0) = T_b$

C.C.4: $T(r,\pi) = T_b$

6)

$$\alpha_1\,\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial \theta}{\partial t}$$

$$\alpha_2\,\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial \phi}{\partial t}$$

C.C.1: $\dfrac{\partial \theta(0,t)}{\partial x} = 0$

C.C.2: $\theta(L_1,t) = \phi(L_1,t)$

C.C.3: $k_1\dfrac{\theta(L_1,t)}{\partial x} = k_2\dfrac{\phi(L_1,t)}{\partial x}$

C.C.4: $\dfrac{\partial \phi(L_1+L_2,t)}{\partial x} = 0$

C.I.: $\theta(x,0) = T_1$

7)

$$\alpha \frac{\partial^2 T}{\partial x^2} + \frac{q_o'''}{\rho c_p} e^{-bx} = \frac{\partial T}{\partial t}$$

C.C.1: $T(0,t) = T_o$

C.C.2: $T(\infty,t) = T_i$

C.I.: $T(x,0) = T_i$

8)

$$\alpha \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] - U \frac{\partial T}{\partial z} + \frac{q'''}{\rho c_p} = 0$$

C.C.1: $\frac{\partial T(0,z)}{\partial r} = 0$, ou $T(0,z) = \text{finite}$

C.C.2: $-k \frac{\partial T(r_o,z)}{\partial r} = h[T(r_o,z) - T_\infty] + \varepsilon\sigma[T^4(r_o,z) - T_{sur}^4]$

C.C.3: $T(r,0) = T_o$

C.C.4: $T(r,\infty) =$ finite

9)

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\frac{\partial T}{\partial z} \right) - \frac{\rho c_p}{k} U \frac{\partial T}{\partial z} = 0$$

C.C.1: $T(-\infty,r) = T_i$

C.C.2: $T(\infty,r) = \text{finite}$

C.C.3: $\frac{\partial T(0,z)}{\partial r} = 0$

C.C.4: $-k \frac{\partial T(r_o,z)}{\partial r} = \begin{cases} 0 & -\infty \le z \le 0 \\ -q_o'' & 0 \le z \le L \\ 0 & z \ge L \end{cases}$

10)

$$\frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 T}{\partial \theta^2} = 0$$

C.C.1: $\frac{\partial T(r,-\pi/2)}{\partial \theta} = 0$

C.C.2: $\frac{\partial T(r,\pi/2)}{\partial \theta} = 0$

C.C.3: $-k\frac{\partial T(R_i,\theta)}{\partial r} = q_i''$

C.C.4: $-k\frac{\partial T(R_o,\theta)}{\partial r} = \begin{cases} 0 & -\pi/2 \le \theta \le 0 \\ h\left[T(R_o,\theta) - T_\infty\right] + \varepsilon\sigma\left[T^4(R_o,\theta) - T_{sur}^4\right] & 0 \le \theta \le \pi/2 \end{cases}$

CAPÍTULO 3

1)

$$\frac{T(x)-T_o}{\frac{q_o'''}{b^2 k}} = 1 - e^{-bL(x/L)} - (bL)(x/L)e^{-bL}$$

$$\frac{T(L)-T_o}{\frac{q_o'''}{b^2 k}} = 1 - (1+bL)\ e^{-bL}$$

2)

$$T(x) = C_1\,x + C_2$$

$$C_1 = -\frac{q_o''}{k}$$

$$q_o'' = h\left[-\frac{q_o''}{k}L + C_2 - T_\infty\right] + \varepsilon\sigma\left[(-\frac{q_o''}{k}L + C_2)^4 - T_{sur}^4\right]$$

$$\mathrm{C_1} = -\frac{19{,}500\ \ (\mathrm{W/m^2})}{15\ (\mathrm{W/m\text{-}^oC})} = -1300\frac{\mathrm{^oC}}{\mathrm{m}} = -1300\frac{\mathrm{K}}{\mathrm{m}}$$

$$\mathrm{C_2} = 761.9\,\mathrm{K}$$

$$T(x) = 761.9\,(\mathrm{K}) - 1300\,(\mathrm{K/m})\,x$$

$$T(0) = 761.9\,\mathrm{K}$$

$$T(L) = 657.9\,\mathrm{K}$$

3)

$$T(r) = C_1 \ln r + C_2$$

$$C_1 = -\frac{q_i''}{k}r_i$$

$$\frac{r_i}{r_o}q_i'' = h\left[-\frac{q_i''}{k}r_i \ln r_o + C_2 - T_\infty\right] + \varepsilon\sigma\left[(-\frac{q_i''}{k}r_i \ln r_o + C_2)^4 - T_{sur}^4\right]$$

$$\mathrm{C_1} = -\frac{35{,}500\ \ (\mathrm{W/m^2})}{3.8\ (\mathrm{W/m\text{-}^oC})}0.055(\mathrm{m}) = -513.82\frac{\mathrm{^oC}}{\mathrm{m}} = -513.82\frac{\mathrm{K}}{\mathrm{m}}$$

$$\mathrm{C_2} = -484.8\,\mathrm{K}$$

$$T(r) = -484.8\,(\mathrm{K}) - \frac{q_i}{k} r_i \ln r$$

$$T(r_i) = -484.8\,(\mathrm{K}) - \frac{35{,}500(\mathrm{W/m^2{-}^oC})}{3.8(\mathrm{W/m{-}^oC})} 0.055(\mathrm{m}) \ln 0.055(\mathrm{m}) = 1005.48\,\mathrm{K}$$

$$T(r_o) = -484.8\,(\mathrm{K}) - \frac{35{,}500(\mathrm{W/m^2{-}^oC})}{3.8(\mathrm{W/m{-}^oC})} 0.055(\mathrm{m}) \ln 0.055(\mathrm{m}) = 604.625\,\mathrm{K}$$

4)

E.D.O: $\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{q'''}{k}\frac{r^2}{r_o^2} = 0$

C.C.1: $\frac{dT(r_i)}{dr} = 0$

C.C.2: $T(r_o) = T_o$

$$C_1 = \frac{q_o''}{5k}\frac{r_i^5}{r_o^2}$$

$$C_2 = T_o + \frac{q_o'' r_o^2}{5k}\left[\frac{1}{4} + \frac{r_i^5}{r_o^5}\right]$$

$$\frac{T(r) - T_o}{\frac{q'' r_o^2}{k}} = \frac{1}{20} + \frac{1}{5}\left[\frac{r_i}{r_o}\right]^5 - \frac{1}{5}\left[\frac{r_i}{r_o}\right]^5 \frac{r_o}{r} - \frac{1}{20}\left[\frac{r}{r_o}\right]^4$$

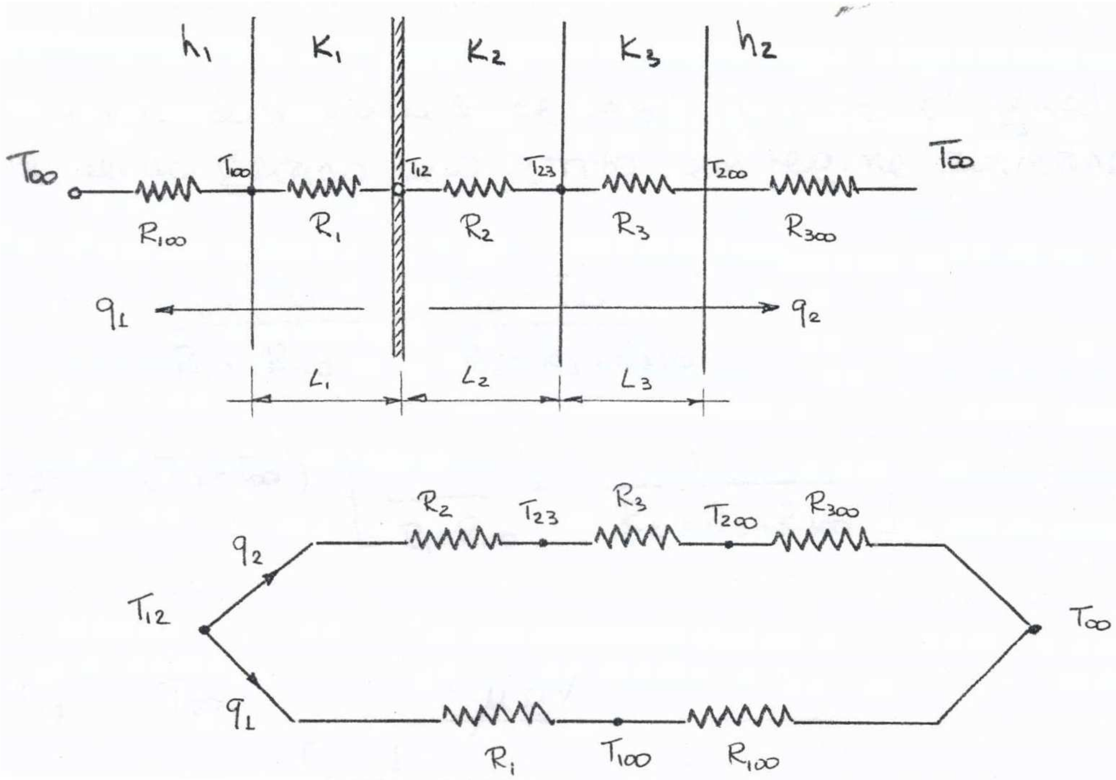
5)

a) $\frac{d^2\theta}{dx^2} - \frac{\rho V c}{k}\frac{d\theta}{dx} - \frac{hP}{kA}\theta = 0$

b) $\omega = \frac{hPL}{\rho c A R}\frac{1}{\ln(\theta_o/\theta_L)}$

6)

a)



b) $q_1 = \dfrac{T_{12} - T_\infty}{R_1 + R_{1\infty}}$ (taxa de calor transferido para o tambor)

$q_2 = \dfrac{T_{12} - T_\infty}{R_2 + R_3 + R_{3\infty}}$ (taxa de calor transferido para a sapata)

c) $T_{12} = T_\infty + \dfrac{\mu p V}{\left[\dfrac{1}{R_1 + R_{1\infty}} + \dfrac{1}{R_2 + R_3 + R_{3\infty}}\right]}$

CAPÍTULO 4

1)

E.D.O.1: $\dfrac{d^2T_1}{dx^2} + \dfrac{q'''}{k} = 0, \qquad 0 \le x \le b$

$$T_1(x) = -\frac{q'''}{2k}x^2 + A_1 x + B_1$$

E.D.O.2: $\dfrac{d^2T_2}{dx^2} - m^2(T_2 - T_\infty) + \dfrac{q'''}{k} = 0, \qquad b \le x \le L$

$$T_2(x) = T_\infty + \frac{q'''}{km^2} + A_2 \sinh mx + B_2 \cosh mx$$

$$m^2 = \frac{hC}{kA_c}$$

C.C.1: $T_1(0) = T_o$

C.C.2: $T_1(b) = T_2(b)$

C.C.3: $\dfrac{dT_1(b)}{dx} = \dfrac{dT_2(b)}{dx}$

C.C.4: $\dfrac{dT_2(L)}{dx} = 0$

$$A_1 = \frac{q'''b}{k} + \frac{\left(T_o - T_\infty + \dfrac{q'''b^2}{2k} - \dfrac{q'''}{km^2}\right)\left(m \sinh mb - m \tanh mL \cosh mb\right)}{\cosh mb - \tanh mL \sinh mb - mb \sinh mb + mb \tanh mL \cosh mb}$$

$$B_1 = T_o$$

$$A_2 = -\frac{\left(T_o - T_\infty + \dfrac{q'''b^2}{2k} - \dfrac{q'''}{km^2}\right)\tanh mL}{\cosh mb - \tanh mL \sinh mb - mb \sinh mb + mb \tanh mL \cosh mb}$$

$$B_2 = \frac{T_o - T_\infty + \dfrac{q'''b^2}{2k} - \dfrac{q'''}{km^2}}{\cosh mb - \tanh mL \sinh mb - mb \sinh mb + mb \tanh mL \cosh mb}$$

2)

E.D.O.1: $\dfrac{d^2T_1}{dr^2} + \dfrac{1}{r}\dfrac{dT_1}{dr} = 0, \qquad R_i \le r \le R_b$

$$T_1(r) = C_1 \ln r + C_2$$

E.D.O.2: $r^2 \dfrac{d^2\theta}{dr^2} + r\dfrac{d\theta}{dr} - \beta^2 r^2 \theta = 0\,, \qquad R_b \le r \le R_0$

$\theta(r) = T_2(r) - T_\infty = C_3 I_0(\beta r) + C_4 K_0(\beta r)$

$\beta^2 = \dfrac{2h}{k\delta}$

$\theta(r) = T_2(r) - T_\infty$

C.C.1: $T_1(0) = T_o$

C.C.2: $T_1(R_b) = T_2(R_b) = \theta(R_b) + T_\infty$

C.C.3: $\dfrac{dT_1(R_b)}{dr} = \dfrac{d\theta(R_b)}{dr}$

C.C.4: $\dfrac{d\theta(R_o)}{dr} = 0$

3)

$\theta = T - T_\infty$

E.D.O.: $\dfrac{d^2\theta}{dx^2} + 2b\dfrac{d\theta}{dx} + m^2\theta = c$

$b = -\dfrac{\rho c_p U}{2k}\,, \quad m^2 = -\dfrac{2h}{kr_o}\,, \quad c = -\dfrac{q'''}{k}$

C.C.1: $\theta(0) = T_o - T_\infty$

C.C.2: $\theta(\infty) =$ finite

$\theta = C_1 \exp(-bx + \sqrt{b^2 - m^2}\, x) + C_2 \exp(-bx - \sqrt{b^2 - m^2}\, x) + \dfrac{c}{m^2}$

$C_1 = 0$ e $C_2 = T_o - T_\infty - \dfrac{c}{m^2}$

$$\frac{T(x) - T_\infty}{T_o - T_\infty} = \left[1 - \frac{q''' r_o}{2h(T_o - T_\infty)}\right] \exp\left[\frac{\rho c_p U r_o}{2k} - \sqrt{\left(\frac{\rho c_p U r_o}{2k}\right)^2 + \frac{2hr_o}{k}}\right]\frac{x}{r_o} + \frac{q''' r_o}{2h(T_o - T_\infty)}$$

4)

E.D.O.: $\dfrac{\rho c_p U}{k}\dfrac{dT}{dx} + \dfrac{2\varepsilon\sigma}{kr_o}(T^4 - T_{sur}^4) = 0$

C.C.: $T(0) = T_o$

$$\frac{2\varepsilon\sigma}{\rho c_p U r_o} x = \frac{1}{4T_{sur}^3} \ln\left|\frac{T_{sur}+T}{T_{sur}-T}\right| + \frac{2}{T_{sur}^3}\tan^{-1}\frac{T}{T_{sur}} + C$$

$$C = -\frac{1}{4T_{sur}^3} \ln\left|\frac{T_{sur}+T_o}{T_{sur}-T_o}\right| + \frac{2}{T_{sur}^3}\tan^{-1}\frac{T_o}{T_{sur}}$$

$$\frac{x}{r_o} = \frac{\rho c_p U}{8\varepsilon\sigma T_{sur}^3}\left\{\ln\left|\frac{1+\frac{T}{T_{sur}}}{1-\frac{T}{T_{sur}}}\right| - \ln\left|\frac{1+\frac{T_o}{T_{sur}}}{1-\frac{T_o}{T_{sur}}}\right| + 8\left[\tan^{-1}\frac{T}{T_{sur}} - \tan^{-1}\frac{T_o}{T_{sur}}\right]\right\}$$

5)

E.D.O.1: $\frac{d^2T_1}{dx^2} - \frac{\rho c_p U}{k}\frac{dT_1}{dx} = 0, \quad -\infty \le x \le 0$

E.D.O.2: $\frac{d^2T_2}{dx^2} - \frac{\rho c_p U}{k}\frac{dT_2}{dx} + \frac{4q_o''}{kd} = 0, \quad 0 \le x \le L$

E.D.O.3: $\frac{d^2T_3}{dx^2} - \frac{\rho c_p U}{k}\frac{dT_3}{dx} = 0, \quad x \ge L$

C.C.1: $T_1(-\infty) = T_i$

C.C.2: $T_1(0) = T_2(0)$

C.C.3: $\frac{dT_1(0)}{dx} = \frac{dT_2(0)}{dx}$

C.C.4: $T_2(L) = T_3(L)$

C.C.5: $\frac{dT_2(L)}{dx} = \frac{dT_3(L)}{dx}$

C.C.6: $T_3(\infty) = \text{finite}$

$$T_1 = A_1 \exp(\rho c_p U x / k) + B_1$$

$$T_2 = A_2 \exp(\rho c_p U x / k) + \frac{4q_o''}{\rho c_p U d} x + B_2$$

$$T_3 = A_3 \exp(\rho c_p U x / k) + B_3$$

$$B_1 = T_i$$

$$A_1 + T_i = A_2 + B_2$$

$$\frac{\rho c_p U}{k} A_1 = \frac{\rho c_p U}{k} A_2 + \frac{4q''_o}{\rho c_p U d}$$

$$A_2 \exp(\rho c_p UL/k) + \frac{4q''_o}{\rho c_p U d} L + B_2 = A_3 \exp(\rho c_p UL/k) + B_3$$

$$A_2 \frac{\rho c_p U}{k} \exp(\rho c_p UL/k) + \frac{4q''_o}{\rho c_p U d} = A_3 \frac{\rho c_p U}{k} \exp(\rho c_p UL/)$$

$$A_3 = 0$$

$$\frac{T_1(x) - T_i}{\dfrac{4kq''_o}{(\rho c_p U)^2 d}} = \exp \frac{\rho c_p UL(x/L)}{k} \left[1 - \exp\left(-\frac{\rho c_p UL}{k}\right) \right]$$

$$\frac{T_2(x) - T_i}{\dfrac{4kq''_o}{(\rho c_p U)^2 d}} = 1 + \frac{\rho c_p UL}{k} \frac{x}{L} - \exp \frac{\rho c_p UL}{k} \left[(x/L) - 1 \right]$$

$$\frac{T_3(x) - T_i}{\dfrac{4q''_o L}{\rho c_p U d}} = 1$$

6)

E.D.O.1: $\frac{d^2 T_1}{dx^2} - \frac{\rho c_p U}{k} \frac{dT_1}{dx} - \frac{4h}{kd}(T_1 - T_\infty) = 0 \qquad 0 < x < \pi R_c$ e $U = \omega R_c$

E.D.O.2: $\frac{d^2 T_2}{dx^2} - \frac{\rho c_p U}{k} \frac{dT_2}{dx} + \frac{4q''_o}{kd} = 0 \qquad \pi R_c < x < 2\pi R_c$ e $U = \omega R_c$

C.C.1: $T_1(0) = T_2(0)$

C.C.2: $\frac{dT_1(0)}{dx} = \frac{dT_2(0)}{dx}$

C.C.3: $T_1(\pi R_c) = T_2(\pi R_c)$

C.C.4: $\frac{dT_1(\pi R_c)}{dx} = \frac{dT_2(\pi R_c)}{dx}$

$$T_1(x) = A_1 \exp\left[-bx + \sqrt{b^2 - m^2} \right] + B_1 \exp\left[-bx - \sqrt{b^2 - m^2} \right] + \frac{c}{m^2}$$

$$T_2 = A_2 + B_2 \exp(\rho c_p Ux/k) + Q_o x$$

$$b = -\frac{\rho c_p U}{2k}, \quad m^2 = -\frac{4h}{kd}, \quad c = -\frac{4h}{kd} T_\infty, \quad Q_o = \frac{4q_o''}{\rho c_p U d}$$

$$A_1 + B_1 + (c/m^2) = A_2 + B_2$$

$$\left[-b + \sqrt{b^2 - m^2}\right] A_1 + \left[-b - \sqrt{b^2 - m^2}\right] B_1 = (\rho c_p UU/k) B_2 + Q_o$$

$$A_1 \exp\left[(-b + \sqrt{b^2 - m^2})\pi R_c\right] + B_1 \exp\left[(-b - \sqrt{b^2 - m^2})\pi R_c\right] + (c/m^2) = A_2 + B_2 \exp(\pi R_c \rho c_p U/k) + \pi R_c Q_o$$

$$A_1(-b + \sqrt{b^2 - m^2}) \exp\left[(-b + \sqrt{b^2 - m^2})\pi R_c\right] + B_1(-b - \sqrt{b^2 - m^2}) \exp\left[(-b - \sqrt{b^2 - m^2})\pi R_c\right] = B_2 (U/\alpha) \exp(\pi R_c \rho c_p U/k) + Q_o$$

7)

E.D.O.1: $\frac{d^2 T_1}{dx^2} + 2b \frac{dT_1}{dx} + m^2 T_1 = c$

$$b = -\frac{\rho c_p U}{2k} \qquad m^2 = -\frac{2h}{k r_o} \qquad c = -\frac{2h}{k r_o} T_\infty$$

E.D.O.2: $\frac{d^2 T_2}{dx^2} - \frac{\rho c_p U}{k} \frac{dT_2}{dx} = 0$

C.C.1: $T_1(-L) = T_i$

C.C.2: $T_1(0) = T_2(0)$

C.C.3: $\frac{dT_1(0)}{dx} = \frac{dT_2(0)}{dx}$

C.C.4: $T_2(\infty) = \text{finite}$

$$T_1 = A_1 \exp(-bx + \sqrt{b^2 - m^2}\, x) + B_1 \exp(-bx - \sqrt{b^2 - m^2}\, x) + \frac{c}{m^2}$$

$$T_2 = A_2 \exp(-2bx) + B_2$$

$$A_1 = \frac{T_i - T_\infty}{\exp(bL - \sqrt{b^2 - m^2}\, L) + \frac{-b + \sqrt{b^2 - m^2}}{b + \sqrt{b^2 - m^2}} \exp(bL + \sqrt{b^2 - m^2}\, L)}$$

$$B_1 = \frac{\frac{-b + \sqrt{b^2 - m^2}}{b + \sqrt{b^2 - m^2}} (T_i - T_\infty)}{\exp(bL - \sqrt{b^2 - m^2}\, L) + \frac{-b + \sqrt{b^2 - m^2}}{b + \sqrt{b^2 - m^2}} \exp(bL + \sqrt{b^2 - m^2}\, L)}$$

$$A_2 = 0$$

$$B_2 = T_\infty + \frac{(T_i - T_\infty)}{\exp(bL - \sqrt{b^2 - m^2}\ L) + \dfrac{-b + \sqrt{b^2 - m^2}}{b + \sqrt{b^2 - m^2}} \exp(bL + \sqrt{b^2 - m^2}\ L)} \left[1 + \frac{-b + \sqrt{b^2 - m^2}}{b + \sqrt{b^2 - m^2}} \right]$$