FORMULÁRIO P3 – TRANSFERÊNCIA DE CALOR E MASSA I

- Balanço de energia: $\dot{E}_e - \dot{E}_s + \dot{E}_g = \dot{E}_{acu}$

- Número de Biot para análise concentrada: $Bi = \frac{hL_c}{k} = \frac{h(V/A_s)}{k} < 0{,}1$

- Comprimento característico: $L_c = \frac{L}{2}$ (parede plana de espessura $2L$), $L_c = \frac{r_o}{2}$ (cilindro longo), $L_c = \frac{r_o}{3}$ (esfera)

- Expressões da análise concentrada com convecção na superfície:

$$t = \frac{\rho Vc}{hA_s}\ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right) \qquad \text{e} \qquad \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

- Transferência de energia na análise concentrada com convecção na superfície:

$$Q = (\rho Vc)(T_i - T_\infty)\left\{1 - \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]\right\}$$

- Adimensionais da análise distribuída:

$$\theta^* = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} \qquad x^* = \frac{x}{L} \qquad r^* = \frac{r}{r_o} \qquad t^* = \frac{\alpha t}{L^2} = \frac{\alpha t}{r_o^2} = \text{Fo}$$

- Solução aproximada para parede plana $(Bi \geq 0{,}1 \text{ e } Fo \geq 0{,}2)$:

$$\theta^* = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)\cos\left(\zeta_1 x^*\right) \qquad \frac{Q}{Q_o} = 1 - \frac{\text{sen}\,\zeta_1}{\zeta_1}\theta_o^*$$

$$Q_o = \rho c V(T_i - T_\infty) \qquad \theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)$$

- Solução aproximada para cilindro longo $(Bi \geq 0{,}1 \text{ e } Fo \geq 0{,}2)$:

$$\theta^* = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right) J_0\left(\zeta_1 r^*\right) \qquad \frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1)$$

$$Q_o = \rho c V(T_i - T_\infty) \qquad \theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)$$

- Solução aproximada para esfera $(Bi \geq 0{,}1 \text{ e } Fo \geq 0{,}2)$:

$$\theta^* = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)\frac{1}{\zeta_1 r^*}\text{sen}\left(\zeta_1 r^*\right) \qquad \frac{Q}{Q_o} = 1 - \frac{3\theta_o^*}{\zeta_1^3}\left[\text{sen}(\zeta_1) - \zeta_1\cos(\zeta_1)\right]$$

$$Q_o = \rho c V(T_i - T_\infty) \qquad \theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 \text{Fo}\right)$$

- Soluções para meio semi-infinito:

CASO 1 – Temperatura na superfície constante: $T(0,t) = T_s$

$$\frac{T(x,t) - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right) \qquad q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

CASO 2 – Fluxo térmico na superfície constante: $q_s'' = q_o''$

$$T(x,t) - T_i = \frac{2q_o''(\alpha t/\pi)^{1/2}}{k}\exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q_o'' x}{k} erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

CASO 3 – Convecção na superfície: $-k\left.\dfrac{\partial T}{\partial x}\right|_{x=0} = h\left[T_\infty - T(0,t)\right]$

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right]\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$q_s''(t) = h\left[T_\infty - T_s(t)\right] \quad erfc(w) = 1 - erf(w)$$

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

w	erf w	w	erf w	w	erf w
0.00	0.00000	0.36	0.38933	1.04	0.85865
0.02	0.02256	0.38	0.40901	1.08	0.87333
0.04	0.04511	0.40	0.42839	1.12	0.88679
0.06	0.06762	0.44	0.46622	1.16	0.89910
0.08	0.09008	0.48	0.50275	1.20	0.91031
0.10	0.11246	0.52	0.53790	1.30	0.93401
0.12	0.13476	0.56	0.57162	1.40	0.95228
0.14	0.15695	0.60	0.60386	1.50	0.96611
0.16	0.17901	0.64	0.63459	1.60	0.97635
0.18	0.20094	0.68	0.66378	1.70	0.98379
0.20	0.22270	0.72	0.69143	1.80	0.98909
0.22	0.24430	0.76	0.71754	1.90	0.99279
0.24	0.26570	0.80	0.74210	2.00	0.99532
0.26	0.28690	0.84	0.76514	2.20	0.99814
0.28	0.30788	0.88	0.78669	2.40	0.99931
0.30	0.32863	0.92	0.80677	2.60	0.99976
0.32	0.34913	0.96	0.82542	2.80	0.99992
0.34	0.36936	1.00	0.84270	3.00	0.99998

- Transferência de energia para condução multidimensional:

$$\left(\frac{Q}{Q_o}\right)_{2D} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right]$$

$$\left(\frac{Q}{Q_o}\right)_{3D} = \left(\frac{Q}{Q_o}\right)_1 + \left(\frac{Q}{Q_o}\right)_2 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right] + \left(\frac{Q}{Q_o}\right)_3 \left[1 - \left(\frac{Q}{Q_o}\right)_1\right]\left[1 - \left(\frac{Q}{Q_o}\right)_2\right]$$